What are memories?

Consider a general system with a smooth configuration space, where each point (state) can be described by a multidimensional vector .

A general transformation of this vector can be defined as

Where is a parameter vector and a vector in the output (inner product) space. The transformation is restricted to be continuous and twice differentiable in . In particular the continuity condition implies:

Is this a memory? It might be if the state of the system remains fixed throughout the memory retrieval process, and in particular if it is fixed in time (). In general physical systems are not stationary for generic states due to system dynamics and noise.

We can get rid of mean system dynamics by requiring the memory to be defined in a stable state of the system:

This condition is naturally met for physical systems in states which are bottom of (appropriate) energy wells. For example, a zero temperature system in an energy minimum. Quenched system dynamics tend to transiently change the initial state to one of these energy minima. Let us thus specialize our transformation to make use of nature:

Where crucially, is now a scalar “energy” function. Stable points of quenched dynamics are minima of this function:

The eigenvalues of the hessian matrix are denoted as are all positive. The index of a square matrix is the number of negative eigenvalues. Therefore at minimum points

A single memory state in a physical systems thus satisfies 3 conditions at once. To include more than one memory we can add another label on the memory states . The 3 conditions above need to apply simultaneously for every memory . is the total number of memories.

Let us impose all of these conditions simultaneously:

In condition we chose norms to get non-negative scalar quantities.

With are inverse temperature-like and enforce the constraints. This can be rewritten as a sum by taking logarithms:

If all constraints are defined as non-negative scalars we can also write: